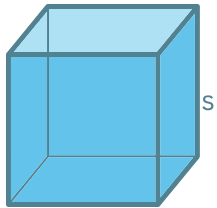# Length, Area and Volume

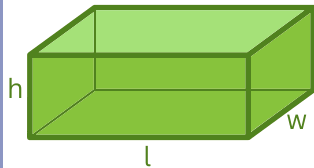## Volume and Surface Area Formulas
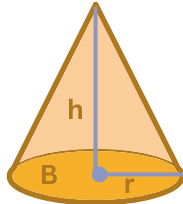
CUBE

$SA = 6s^2$
$V = s^3$

RECTANGULAR PRISM

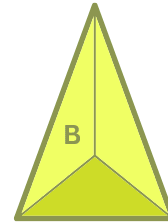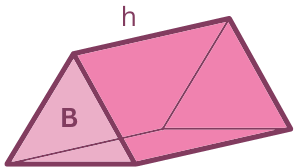$SA = 2(lw + lh + wh)$
$V = lwh$

CONE

$SA = \pi rs + \pi r^2$
$V = \frac{1}{3}Bh$

TRIANGULAR PYRAMID

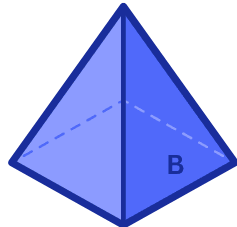SA = sum of face areas
$V = \frac{1}{3}Bh$

TRIANGULAR PRISM
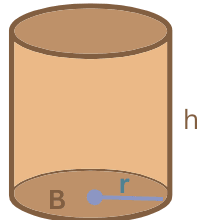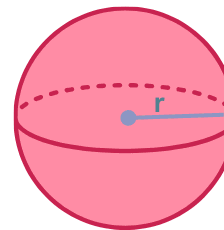
$SA = 2B + 2P$
$V = Bh$

SQUARE PRISM

SA = sum of face areas
$V = \frac{1}{3}Bh$

CYLINDER

$SA = 2\pi r + 2\pi r^2$
$V = Bh$

SPHERE

$SA = 4\pi r^2$
$V = 4/3\ \pi r^3$

Learning Intention

Topic: Perimeter

6A

1 I can convert between metric units of length.

e.g. Convert these measurements to the units shown in the brackets.
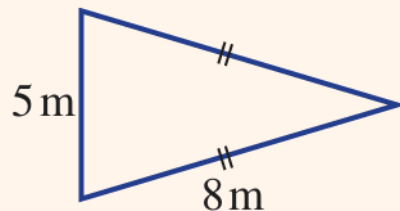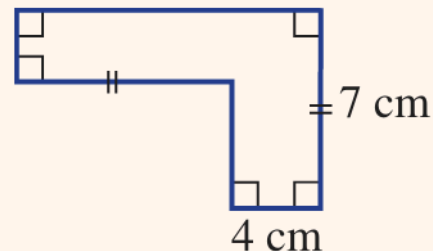
a 9.2 cm (mm)   **b** 61 000 cm (m)

6A

2 I can find the perimeter of basic shapes, including composite shapes.

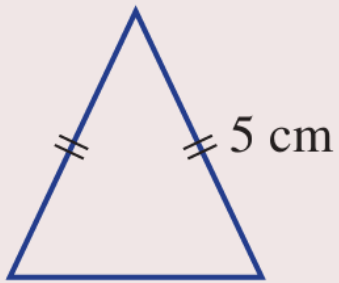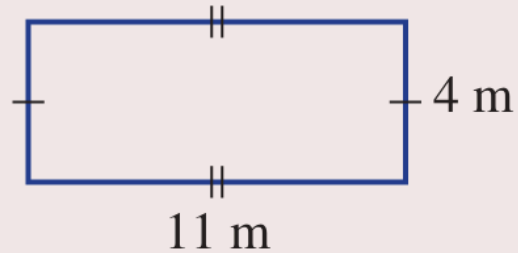e.g. Find the perimeter of these shapes.

a

5 m

8 m

b

7 cm

4 cm

Perimeter

What's perimeter?

- Perimeter of a shape is the distance around the shape

- Sides with the same markings are of equal **length**

5 cm

3 cm

$P = 2 \times 5 + 3$
$= 13$ cm

4 m

11 m

$P = 2 \times 11 + 2 \times 4$
$= 30$ m

$\div 10$ $\div 100$ $\div 1000$

mm   cm   m   km

x10   x100   x1000
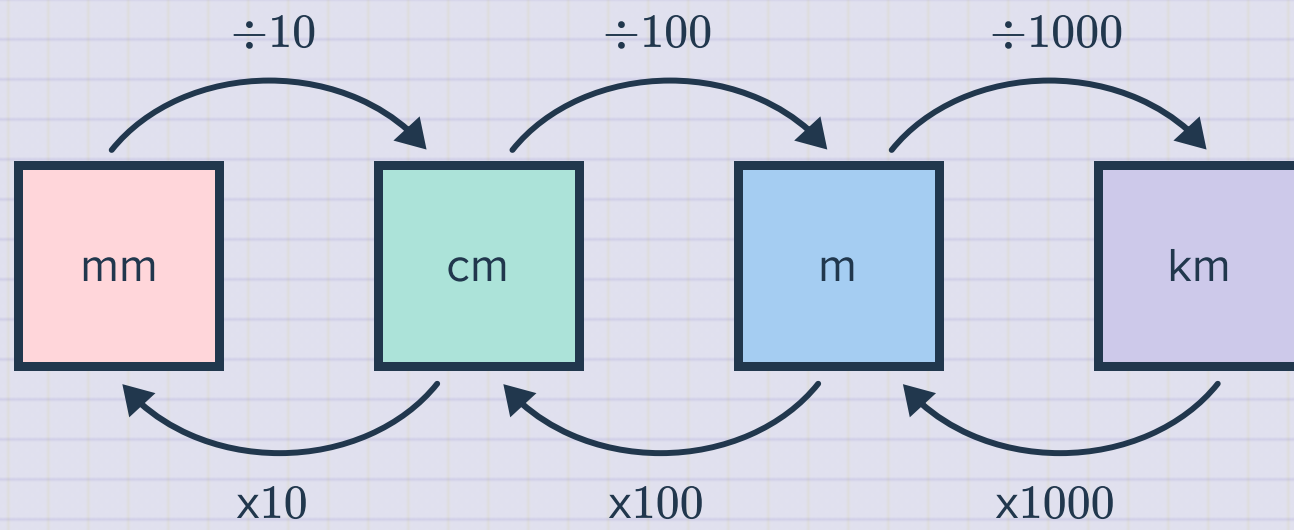
Perimeter of Composite Shapes

- What's a composite shape?
    - any shape that is made up of two or more shapes

- We see what lengths are on the outside - some sides may now be shared

- Composite shape in real life:
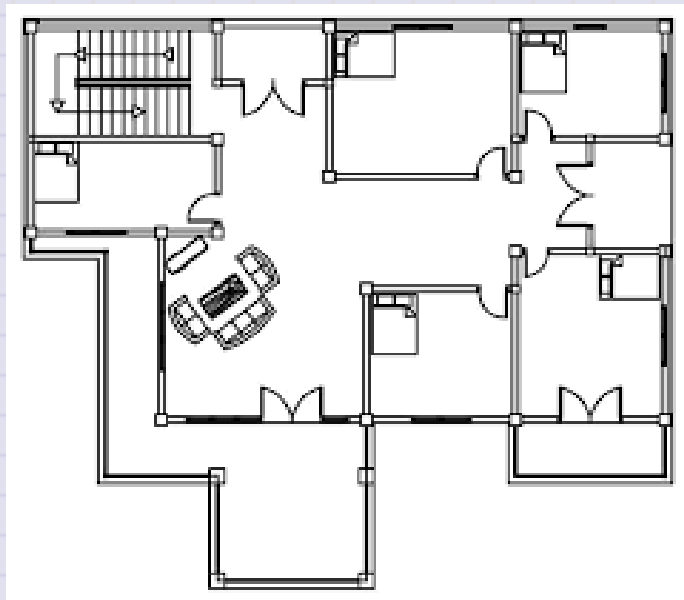
Learning Intention

Topic: Circumference and Sector Perimeters

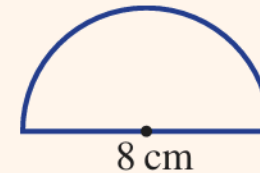|		
6B	**3 I can find the circumference of a circle.** e.g. Find the circumference of a circle with a diameter of 5 m, correct to two decimal places.	☐
6B	**4 I can find the perimeter of a semicircle or a quadrant.** e.g. Find the perimeter of this semicircle correct to two decimal places. 8 cm	☐

Parts of a circle: distances

- **Circumference**: perimeter of a circle
    - remember: perimeter: distance around a shape
- **radius**: distance from centre of circle to its edge
    - 🤔: How many radii can there be?
- **diameter**: distance from one end of the circle to the other, through the centre
    - diameter = 2 × radius   (🤔 why?)
- **sector**: part of a circle marked off by an arc and its two radii
   - a fraction of the circle
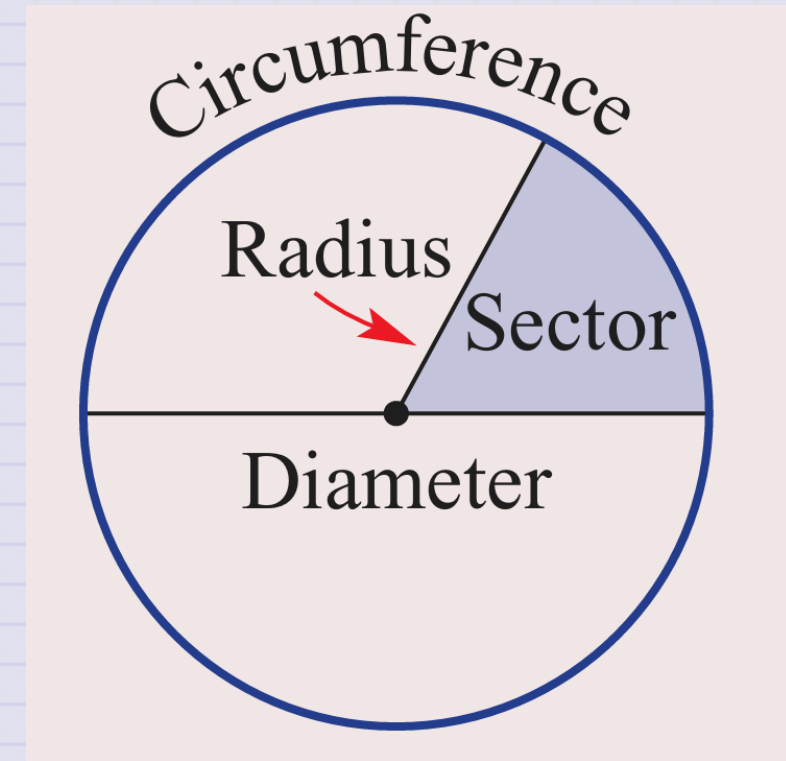
Introducing: Pi π

- This is an irrational number: goes on forever, but approximately 3.14

- Archimedes approximated pi by drawing 2 polygons with *96* sides: inside a circle and outside

- Universal constant: relationshp between diameter and circumference

π is on your calculator:

- Let's find it now

- in most calculators, it's on the *bottom* near the *right* of the numbers ⬊

Circumference of Circle
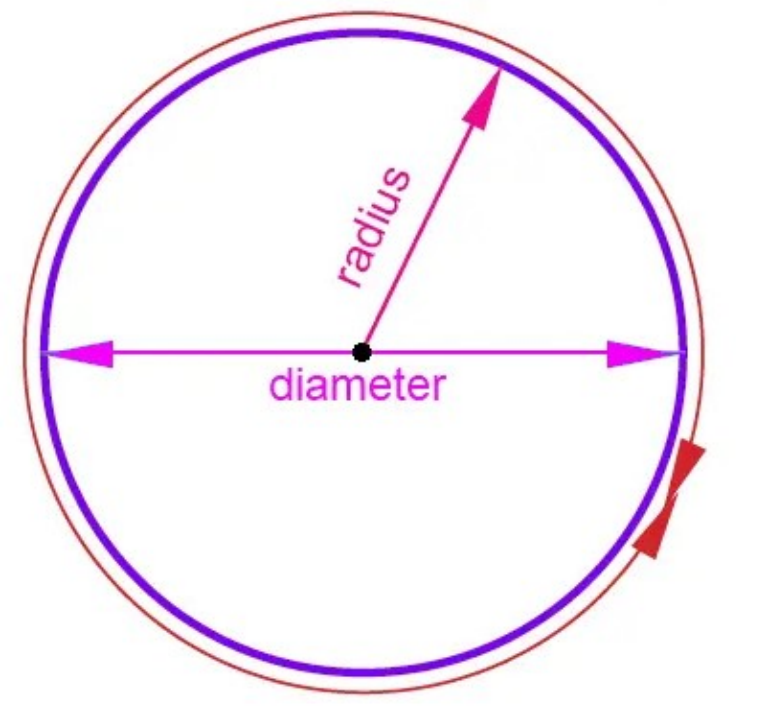
Circumference Formula

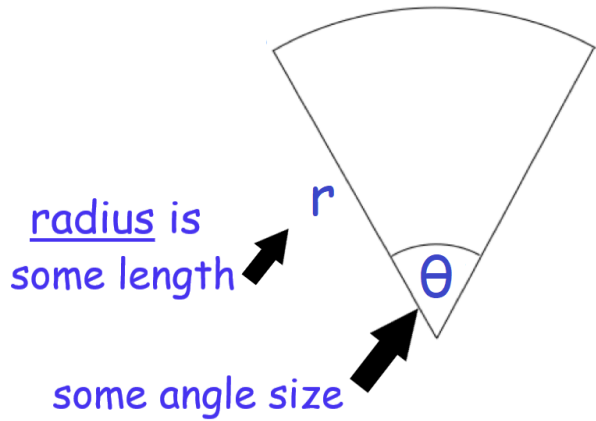$$\text{Circumference} = d \times \pi$$
$$\text{Circumference} = 2 \times \pi \times r$$

Where, d = diameter, r = radius

(remember we said earlier: diameter = $2 \times$ radius)

and $\pi \approx 3.14$

Perimeter of a Sector

- Remember: Circumference = $\pi \times 2 \times r$

- $$\text{Arc-length} = \frac{\theta}{360°} \times \pi \times 2 \times r$$

- $$\text{Perimeter} = \text{arc-length} + 2 \times r$$

- where r is the radius and θ is the angle of the sector

  Note: θ is not 0: it is a Greek symbol that just looks similar

Special sectors

- A half circle is called a **semicircle**

- A quarter circle is called a **quadrant**

Learning Intention

Topic: Area of Quadrilaterals and Triangles

6C

5 I can convert between metric units of area.
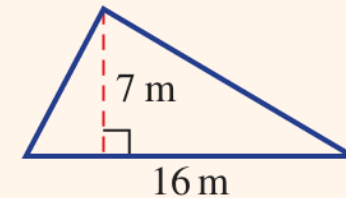e.g. Convert these measurements to the units shown in the brackets.
a 5.32 cm^2 (mm^2)   **b** $728\,000 \text{ cm}^2$ (m^2)

6C

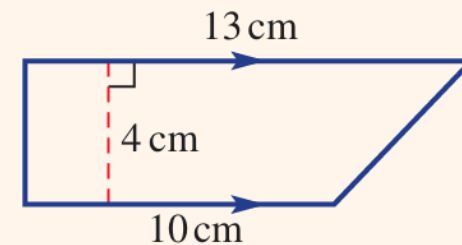**6 I can find the area of rectangles, triangles and parallelograms.**
e.g. Find the area of this triangle.

7 m
16 m

6C

7 I can find the area of rhombuses and trapeziums.
e.g. Find the area of this trapezium.
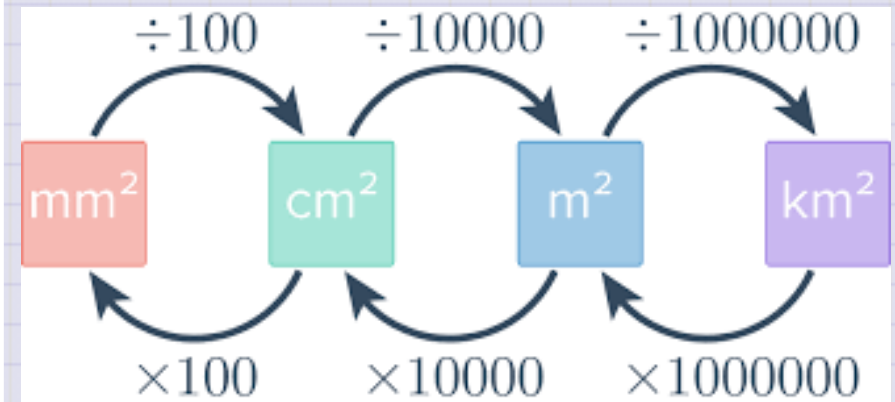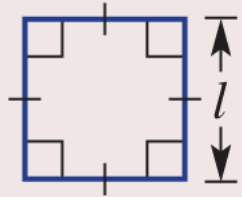
13 cm
4 cm
10 cm

Area of Quadrilaterals and Triangles

Quick check: What's an area?

- The area of a two-dimensional shape is a measure of the space enclosed within its boundaries.
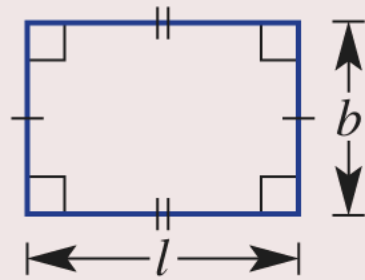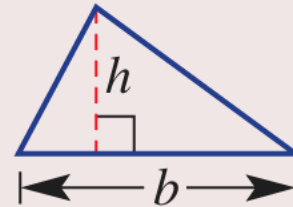
Square

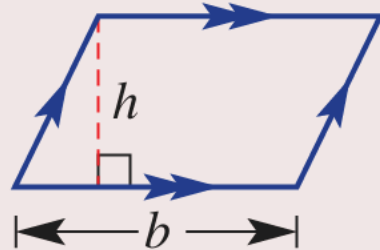$$\text{Area} = l^2$$

Rectangle

$$\text{Area} = lb$$

Triangle
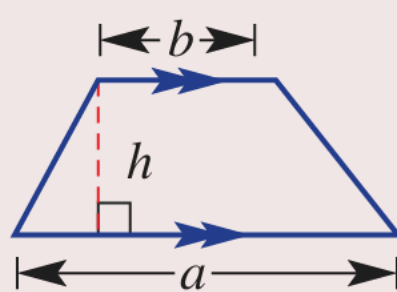
$$\text{Area} = \tfrac{1}{2}bh$$
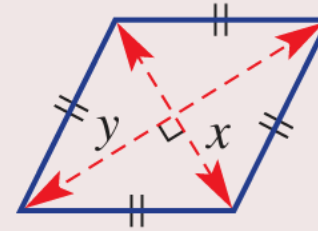
Parallelogram

$$\text{Area} = bh$$
(Can also be used for a rhombus)

Trapezium
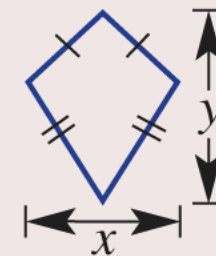
$$\text{Area} = \tfrac{h}{2}(a + b)$$

Rhombus

$$\text{Area} = \tfrac{1}{2}xy$$
(Can also be used for a square)

Kite

$$A = \tfrac{1}{2}xy$$

Note: The height (h) in the formulas for the area of a triangle, parallelogram and trapezium must be perpendicular (at 90∘) to the base.
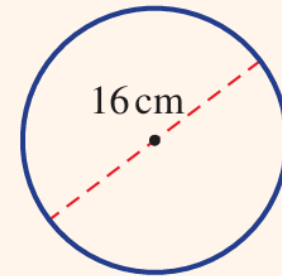
Learning Intention

Topic: Area of a circle

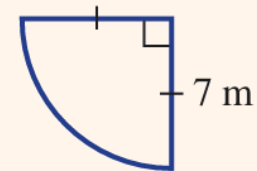**6D** 8 **I can find the area of a circle.**
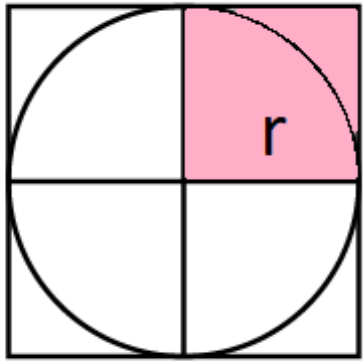e.g. Find the area of this circle correct to two decimal places.

16 cm

6D 9 **I can find the area of a quadrant or semicircle.**
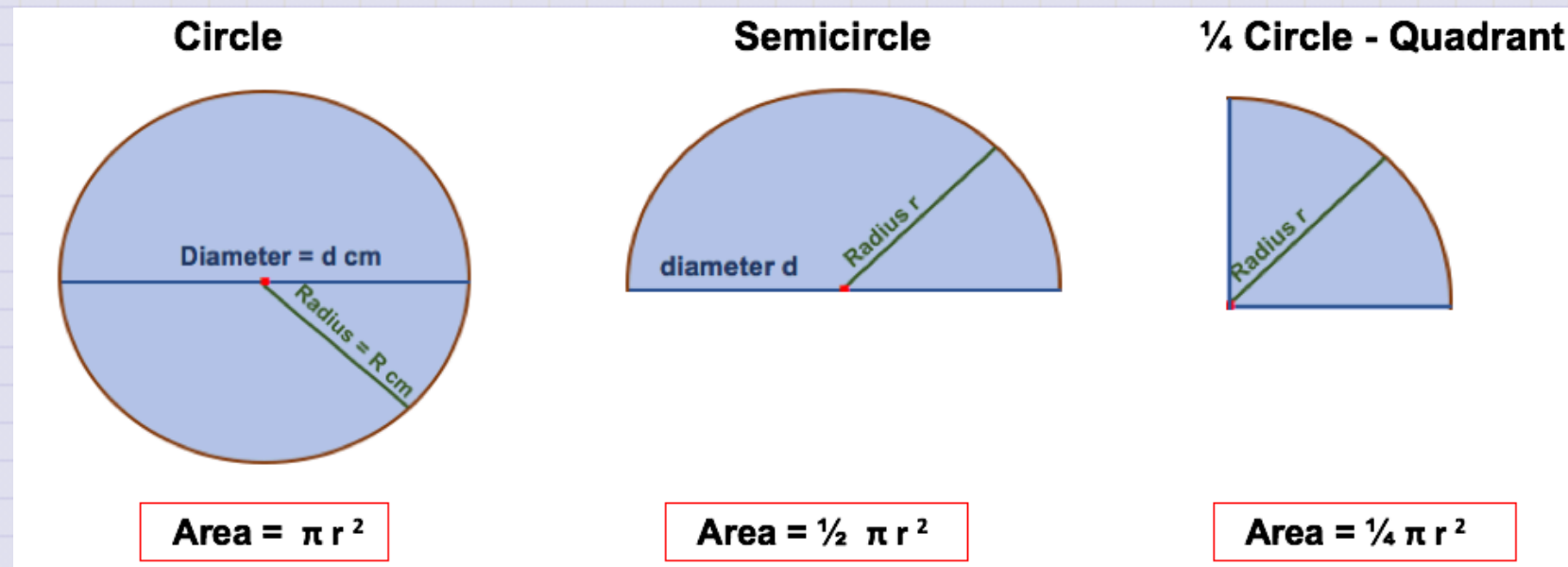e.g. Find the area of this quadrant correct to two decimal places.

7 m

a bit less than

4× ⬛

Area of a Circle

- And guess what, $\pi \approx 3.14$ is a bit less than 4 too!

- $$\text{Area of circle} = \pi \times r^2$$

- where r is the radius and $\pi \approx 3.14$

What if we have a fraction of a circle?

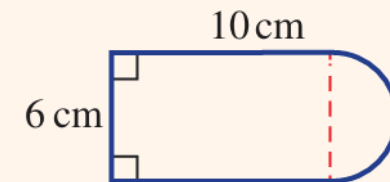| Circle	Semicircle	¼ Circle - Quadrant
Diameter = d cm, Radius = R cm	diameter d, Radius r	Radius r
Area = πr^2	Area = $\frac{1}{2} \pi r^2$	Area = $\frac{1}{4} \pi r^2$

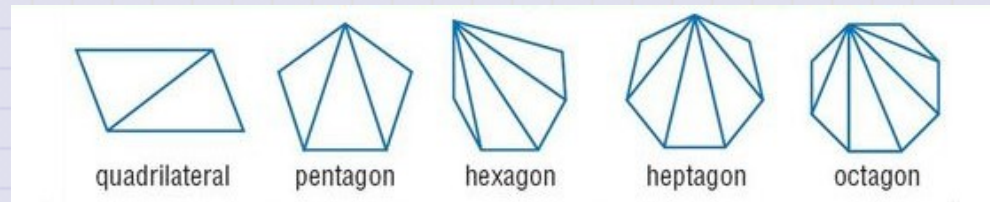# Learning Intention

Topic: Composite Shapes

6E

10 **I can find the perimeter and area of simple composite shapes.**
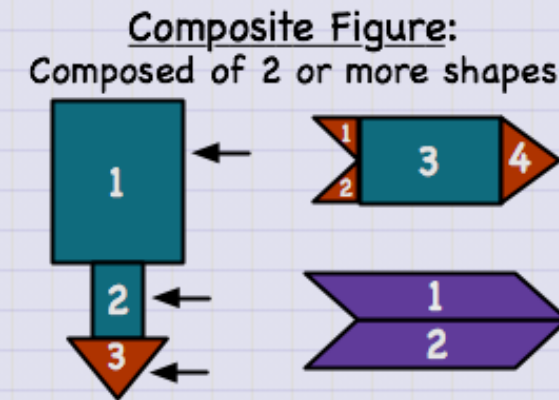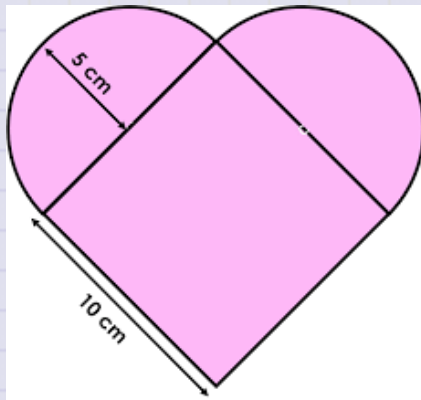e.g. Find the perimeter and area of this composite shape correct to two decimal places.

10 cm

6 cm

Composite Shapes

Composite shape: any shape that is made up of two or more geometric shapes

For example: Any polygon can be broken down into triangles

- This fact is used by GPUs (Graphical Processing Units) to create graphics for video games

Perimeter and Area of Composite Shapes

1. To find the perimeter or area of a composite shape, we break it down into shapes we know (example)

2. a) Perimeter: find the perimeter of the individual shapes and subtract shared sides
   b) Area: Find the area of the individual shapes
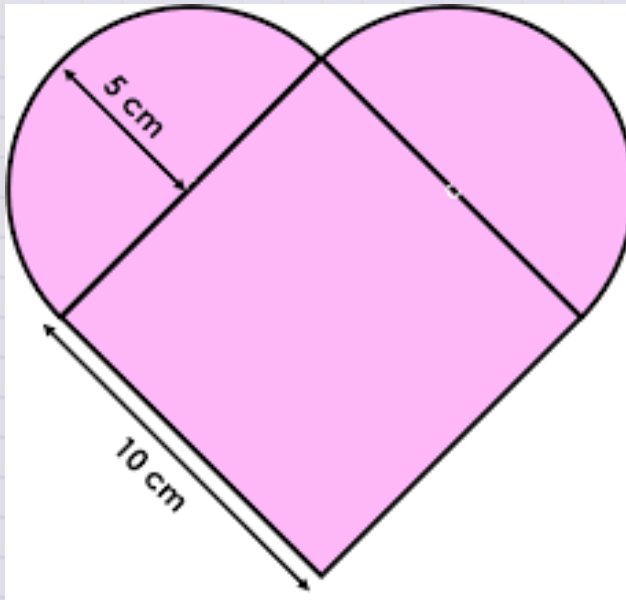
Steps to Find Areas of Composite Shapes

This can be by adding *or* subtracting shapes

- Divide shape diagram with lines (no overlapping shapes)
- Label sides/heights with dimensions
- Calculate areas (give them names like A_1)
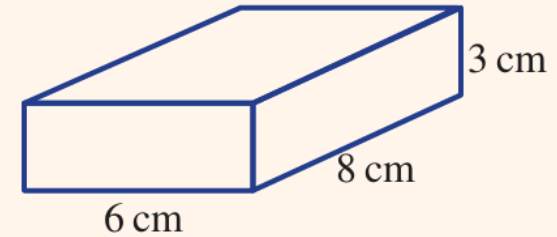- Add up all areas to find the total
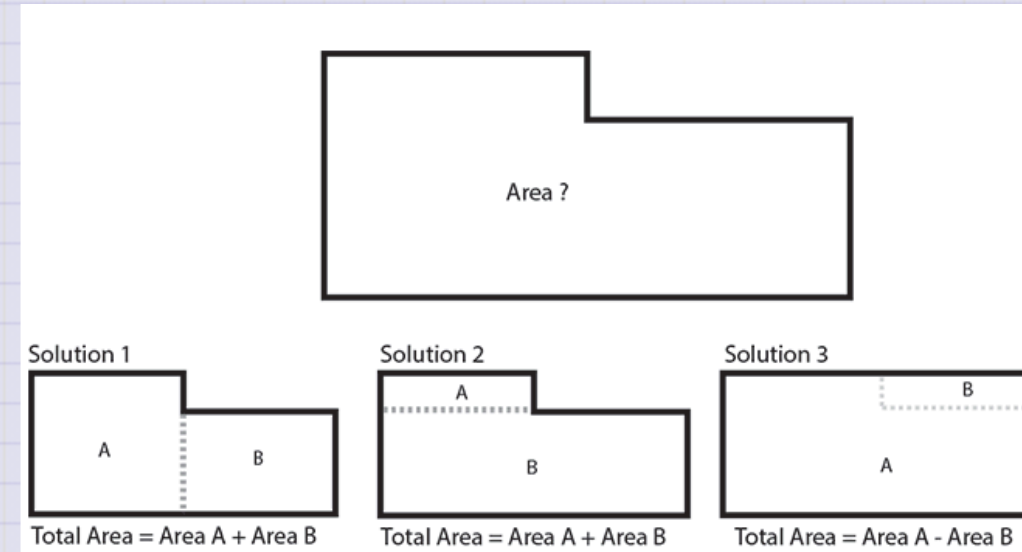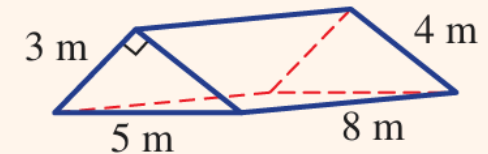
Learning Intention

Topic: Surface Area of Prisms

6F

11 I can find the surface area of a rectangular prism.
e.g. Find the surface area of this rectangular prism.

3 cm
8 cm
6 cm

6F

12 I can find the surface area of a triangular prism.
e.g. Find the surface area of this triangular prism.

3 m
4 m
5 m
8 m

Prisms

What is a prism?

- 3d shape: solid

- cross-section is a **polygon**
  - cross-section: what you get when you slice a shape like bread

Prism: a solid with a **uniform polygonal cross-section**

Prisms in real life

Prisms are named after their cross-section, e.g. triangular, pentagonal

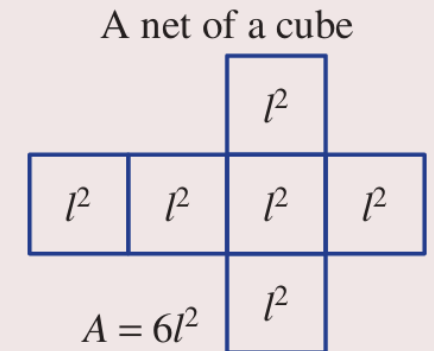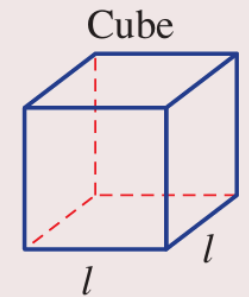What kind of prisms are these?

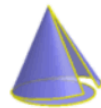- 🤔: What's another name for a square prism? For a rectangular prism?

Surface Area of Prisms

To find the surface area of a prism, we break it down into its **net**

Net: a 2D illustration of all the surfaces of a solid

Cube

l

l

l

A net of a cube

l^2

l^2 l^2 l^2 l^2

$A = 6l^2$

l^2

Nets of the Cube
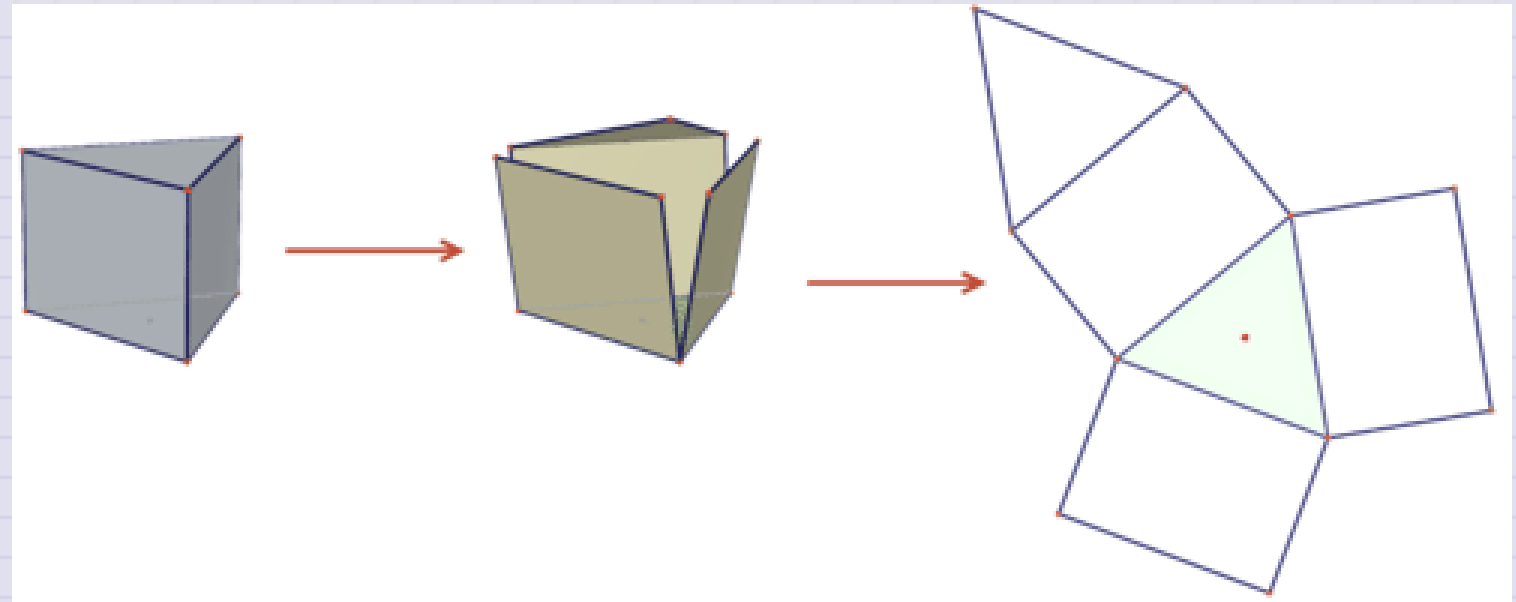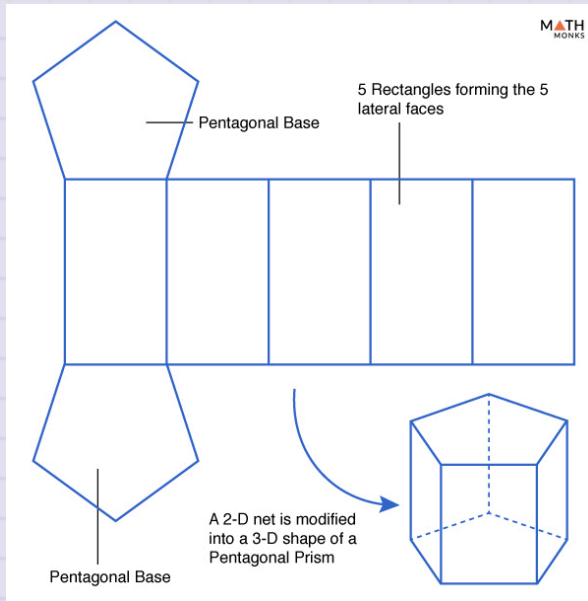
Net of a Cone

Net of a Cylinder

Net of a Square Based Pyramid

How to draw a net
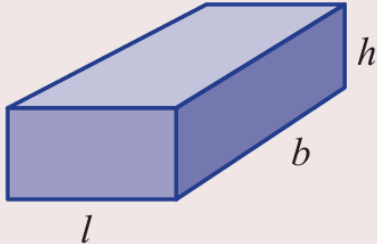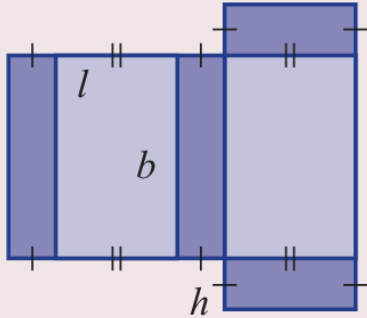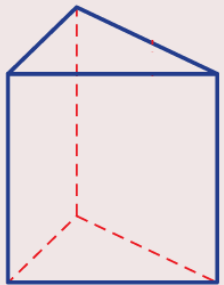
Generally, each prism is made up of:
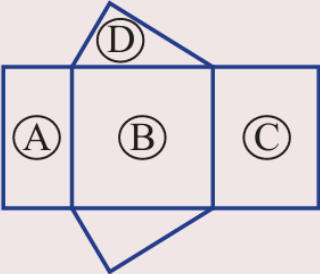
- a bottom and top (the cross-section)

- rectangles connecting these

- You can start with either a rectangle or the base

- Count sides to make sure you've got them all

Finding the Surface Area

- So you have the net, now what?

- Add up all the areas of the shapes in the net

Solid	Net	Surface area
Rectangular prism		$A = 2(lb + bh + lh)$
Triangular prism		$A = \text{area}\,\text{Ⓐ} + \text{area}\,\text{Ⓑ} + \text{area}\,\text{Ⓒ}$ $+ 2 \times \text{area}\,\text{Ⓓ}$

Learning Intention

Topic: Surface Area of Cylinders

6G

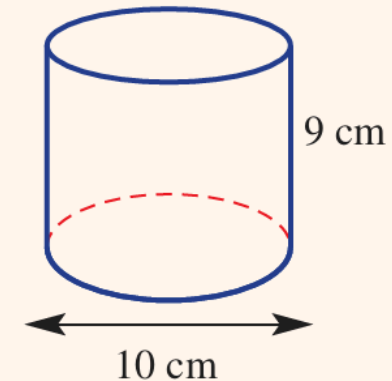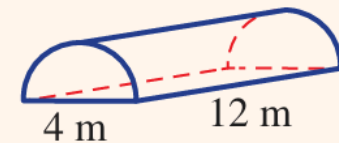**13 I can find the surface area of a cylinder.**
e.g. Find the surface area of this cylinder correct to two decimal places.

9 cm

10 cm

6G

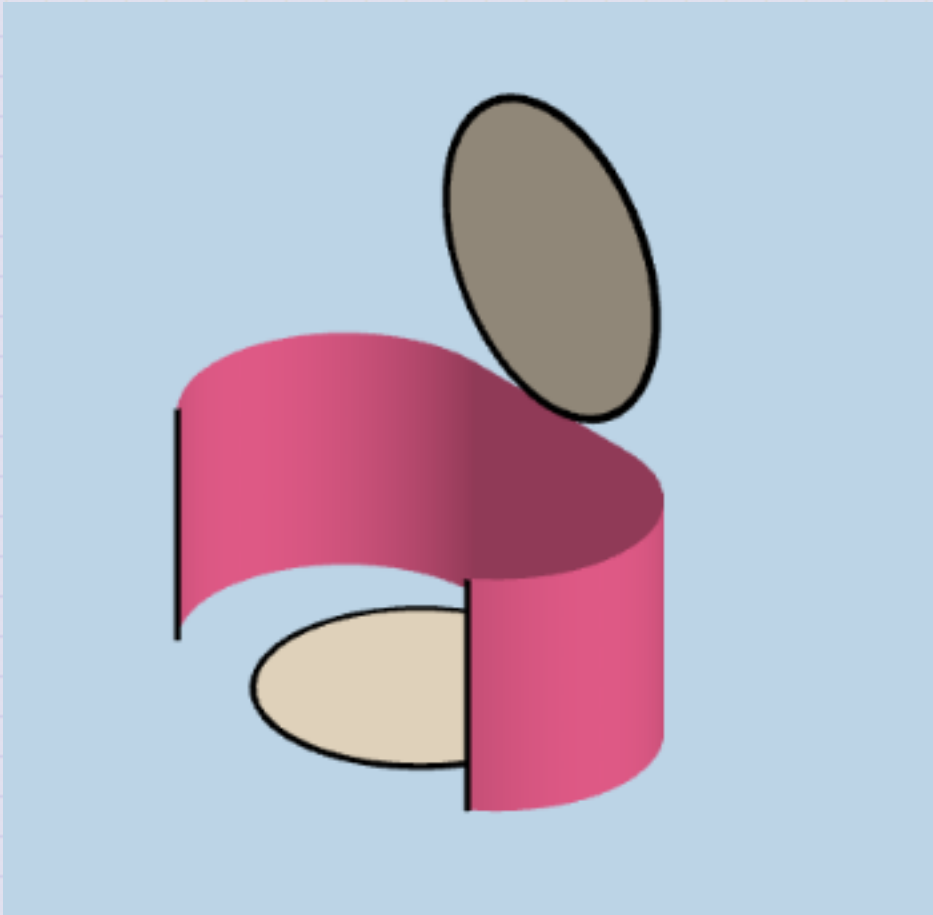**14 I can find the surface area of a cylindrical portion.**
e.g. Find the surface area of this half cylinder correct to two decimal places.

4 m   12 m

What is a **cylinder**?

- Like a circular 'prism': with a uniform circular cross-section

Surface Area

Formula

- Area of the circles = πr^2 each

- Area of the "tube" = $2\pi rh$ (Why?)

- (We can unfold it into a rectangle with the circumference as a side)

- Put together, we get:

- $$A = 2\pi r^2 + 2\pi rh$$

- 5-11 Volume of a prism

Volume of Prisms

- 5-13 Volume and capacity

Capacity

What is capacity? What does it mean for a bottle to have more capacity?

- **Capacity**: the amount of fluid a container can hold
  - usually measured in litres or millilitres
  - also measured in kilolitres and megalitres

Real-life Examples of Capacity

milliLitre	Litre	kiloLitre	MegaLitre
1ml = an eyedropper	Milk comes in 1 Litre	4 full bathtubs = 1 kiloLitre	An Olympic swimming pool = 2.5 MegaLitres
center	center		

- What's the capacity of your water bottles?

Converting between units

Remember: a millilitre is smaller than a litre, so a big number of millilitres will be a small number of litres, not the other way around

1 L = 1000 mL

1 kL = 1000 L

1 ML = 1000 kL

    = 1,000,000 L   (1 million litres!)

middleright

Converting to Capacity from Volume

bottomcorner

We can also convert from volume to capacity

1 mL = 1 cm^3

1L = 1000 cm^3

1 kL = 1 m^3